Environmental Policy in an International Duopoly

Feenstra, T.L.; Kort, P.M.; de Zeeuw, A.J.

Publication date:
1997

Citation for published version (APA):
Feenstra, T. L., Kort, P. M., & de Zeeuw, A. J. (1997). *Environmental Policy in an International Duopoly: An Analysis of Feedback Investment Strategies*. (CentER Discussion Paper; Vol. 1997-43). Microeconomics.

Environmental policy in an international duopoly: an analysis of feedback investment strategies.

Talitha Feenstra, Peter Kort

Aart de Zeeuw*

Department of Economics and CentER, Tilburg University, The Netherlands.

April 8, 1997

Abstract

This paper discusses environmental policy instruments in a differential-game model of international trade with oligopolistic competition. Strategic interactions occur if firms use feedback strategies and therefore react on decisions of their competitor. Eventually this harms firm profits, because all firms act strategically.

A firm reacts differently if its competitor is subject to an environmental standard than if it is subject to an environmental tax. Under open-loop investment strategies and feedback strategies of energy use, environmental taxes always give rise to more investment for strategic reasons than standards. This confirms results of multistage static models of the same problem. The new result is that under feedback investment strategies the reverse can be the case.

Key words: Environmental policy competition; Duopoly; Differential game

JEL classification: C73; D92; Q48

*Research for this paper was sponsored by the Netherlands Organization for Scientific Research. We like to thank Rob Aalbers, Sjak Smulders, Piet Verheyen, and the participants of the 6^{th} EAERE Conference, Umeå, Sweden and the NWO workshop on international environmental issues, Ede, the Netherlands, where previous versions were presented, for stimulating discussions. We also thank Chaim Fershtman for his comments.

1 Introduction

Flexibility is an important reason to advocate economic environmental policy instruments like emission taxes. More than under traditional command and control measures, polluters can choose to what degree and in which way they decrease their pollution. Consequently, the regulator may reach a high allocative efficiency, even with inadequate information about the cost structure of the polluting firms.

In case of imperfect competition, however, flexibility has also another effect, because in such markets commitment plays an important role (see e.g., Tirole, 1988). In some cases it is advantageous for a firm in oligopoly to be able to 'burn its bridges'. The ability to bind itself to certain actions gives the firm a relatively strong position towards a competitor. More flexibility by its nature decreases the possibilities to commit.

The characteristics of government policy influence the flexibility of a firm. If government policy is implemented by rigid prescriptions, it is a commitment for the firm. If, on the contrary, the government bases its policy on incentives, firms are more flexibile so that they have less commitments. Brander and Spencer (1983) analyzed the differences between trade policies when trade can be characterized as an international oligopoly. If governments want high profits for their home firms, they may want to provide home firms with commitment via their trade policy.

In a similar way, environmental policy instruments may affect the international competitiveness of firms. It was found that environmental standards are unambiguously 'better' than taxes, in a multistage static model of international rivalry where allocative efficiency can be neglected (Ulph, 1992). Under standards, firms gain commitment, so that they earn more profits, while regulators reach the same environmental target.

This paper shows that this conclusion depends on the type of investment strategy applied by firms. In a multistage static model, investment decisions are taken once and for all and that is another type of commitment for firms. In order to relax this implicit assumption, the richer framework of differential games is needed. In that framework, one can distinguish different types of strategies. In particular, an open-loop decision strategy means that the firms choose their actions at the beginning of the planning period and stick to these actions no matter what happens. This implies a relatively large commitment, similar to the multistage static model. A feedback decision strategy, though, means that the firms condition their actions on observations on the current state of affairs. This implies that firms are not committed and can react indirectly on each other's past actions.

It will be shown that the result that standards are 'better' than taxes is ambiguous and depends on the type of investment strategy applied by the firm. To be more precise, for open-loop investment strategies the results from multistage modelling are confirmed, but for feedback investment strategies it is ambiguous whether standards or taxes are 'better'. Feedback strategies lead to more strategic interaction, which drives the firms to higher investment with lower profits. Under taxes this effect is mitigated due to the substitution between energy and capital. That does not occur under standards, because in that case firms are at a corner solution for their energy use and do not substitute between energy and capital. Therefore, the use of standards reduces over-investments, which improves profits, as was found before, but it also increases over-investments, which reduces profits, since the mitigating effect is absent. The net effect can go both ways.

Section 2 presents international rivalry as a differential game model of two competing firms. Section 3 derives the equilibrium to this model under feedback investment strategies. A comparison of the equilibrium under taxes and standards gives the main result of the paper. The economic explanation for this result is provided at the end of this section. In section 4 we discuss choices of parameter values and how results depend on these values. Section 5 concludes the paper.

2 An international duopoly model

The model describes a duopoly where each competitor is situated in a different country[1] and faces the environmental policy of that country. The decision makers are firms and regulators. The regulator in each country has a fixed environmental target $\overline{e}_i(t)$ for all times t and prefers high profits for its home firm. Emissions are assumed to have local effects only. Firms and countries are connected through the common market for outputs. Each firm is a profit maximizer. Throughout the whole paper perfect foresight is assumed.

Environmental policy consists either of a tax on energy or of a standard on the use of energy. It is assumed that emissions are directly related to energy use, which implies that an emission tax or standard is equivalent to an energy tax or standard. The regulator sets the level of the tax or the standard beforehand. Taxes, τ^i, are set such that, for all t, $e^i(t) = \overline{e}^i(t)$. The regulator is assumed to have full information on the firm so that it can accomplish this. Standards are simply equal to $\overline{e}^i(t)$. Under these assumptions, taxes and

[1] With equal environmental policy instruments in each country, the results also apply to a duopoly in one country.

standards result in the same energy use.

Each firm i maximizes its discounted stream of profits $\int_0^\infty e^{-rt}\pi^i(t)dt$, where r is a constant rate of discount. Let x^i denote domestic and x^j foreign output, p^e the price of energy, I^i investments and $C(I^i)$ costs of investments. Firm i's profits, $\pi^i(t)$, are given by:

A. In case of a tax

$$\pi^i(t) = R^i(x^i(t), x^j(t)) - (p^e(t) + \tau^i(t))e^i(t) - C(I^i(t)) \tag{1}$$

B. In case of a standard

$$\pi^i(t) = R^i(x^i(t), x^j(t)) - p^e(t)e^i(t) - C(I^i(t)), \tag{2}$$

where in this last case $e^i(t)$ must be smaller than or equal to $\overline{e}^i(t)$.

It will be assumed that revenues, R^i, are concave in outputs and furthermore satisfy: $\frac{\partial^2 R^i}{\partial x^i \partial x^j} < 0$, and $\frac{\partial R^i}{\partial x^j} < 0$. Output is a function of the two production factors energy and capital, $x^i = x^i(e^i, K^i)$. It is assumed that the production function satisfies $\frac{\partial x^i}{\partial e^i} > 0$, $\frac{\partial x^i}{\partial K^i} > 0$, $\frac{\partial^2 x^i}{\partial e^{i2}} < 0$, $\frac{\partial^2 x^i}{\partial K^{i2}} < 0$, $\frac{\partial^2 x^i}{\partial e^{i2}}\frac{\partial^2 x^i}{\partial K^{i2}} < (\frac{\partial^2 x^i}{\partial e^i \partial K^i})^2$ and $\frac{\partial^2 x^i}{\partial K^i \partial e^i} > 0$. The last condition means that more capital increases the marginal productivity of energy. Firms must decide on outputs and investments or, which is equivalent, on energy use and investments.

The costs of investment, $C(I^i)$, include adjustment costs and acquisition costs. The cost function is assumed to be increasing and convex: $C'(I^i) > 0$, $C''(I^i) > 0$. Zero investment involves no costs: $C(0) = 0$. Investments add to the capital stock according to the standard equation for capital accumulation:

$$\dot{K}^i(t) = I^i(t) - d^i K^i(t), \tag{3}$$

where d^i is the constant rate of depreciation. Summarizing, the duopoly is modelled as the following differential game.

A. In case of taxes in both countries:

$$\max_{I^i, e^i} \int_0^\infty e^{-rt}\{R^i(x^i(t), x^j(t)) - (p^e(t) + \tau^i(t))e^i(t) - C(I^i(t))\}dt \tag{4}$$

$$\text{s.t. } \dot{K}^i(t) = I^i(t) - d^i K^i(t) \tag{5}$$

$$e^i(t), I^i(t) \geq 0 \qquad i = 1, 2. \tag{6}$$

B. In case of standards in both countries:

$$\max_{I^i, e^i} \int_0^\infty e^{-rt}\{R^i(x^i(t), x^j(t)) - p^e(t)e^i(t) - C(I^i(t))\}dt \quad (7)$$

$$\text{s.t.} \;\; \dot{K}^i(t) = I^i(t) - d^i K^i(t) \quad (8)$$

$$e^i(t) \leq \overline{e}^i(t) \quad (9)$$

$$e^i(t), I^i(t) \geq 0 \qquad i = 1, 2. \quad (10)$$

C. In case of a standard in country i and a tax in country j the differential game is a combination of A and B.

3 Equilibrium and economic interpretation

3.1 Equilibrium strategies of energy use

Since energy use does not appear in the system dynamics, choices of a certain level of energy have no effect on future periods. Therefore it is possible to solve for the e^i at each time separately as a function of K^i, K^j, τ^i and τ^j (or $\overline{e}^i$ and $\overline{e}^j$). Assuming that each firm takes the energy input of the other firm as given, the first order conditions for an optimal choice of e^1 and e^2 can be formulated. These are the usual equilibrium conditions on marginal benefits and marginal costs.

A. In case of taxes in both countries:

$$\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial e^i} = \tau^i + p^e, \qquad i = 1, 2. \quad (11)$$

B. In case of standards in both countries:

$$\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial e^i} \geq p^e \quad (12)$$

$$e^i(t) \leq \overline{e}^i(t) \quad (13)$$

$$\left(\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial e^i} - p^e\right)(\overline{e}^i - e^i) = 0, \qquad i = 1, 2. \quad (14)$$

C. In case of a standard in country i and a tax in country j, for country i equations (12) to (14) must be satisfied while for country j equation (11) applies.

These conditions result in Nash equilibrium levels of energy use, $e^{iN}(K^i, K^j)$. It is assumed that in equilibrium both firms are in the market. From (11) or (12) it follows that in the equilibrium $\frac{\partial R^i}{\partial x^i} > 0$. This is reasonable since it is never optimal for a firm to have an output where marginal revenues are negative.

If the two firms cooperate to maximize joint profits, the first order conditions for energy use in case of taxes in both countries are:

$$\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial e^i} + \frac{\partial R^j}{\partial x^i}\frac{\partial x^i}{\partial e^i} = \tau^i + p^e, \qquad i = 1,2 \tag{15}$$

which is a modification of (11). In this case effects on foreign profits are included in the marginal benefits. In case of standards, first order conditions are a similar modification of (12) to (14). The resulting optimal levels of energy use are denoted by $e^{iC}(K^i, K^j)$.

3.2 Equilibrium strategies of investment

In this model, firms are in a dynamic environment where they have to make decisions over a longer period of time. Different strategies can be distinguished. If firms apply so called open-loop strategies, they do not react on current state variables. Open-loop investment strategies are a function of time only, $I^{iOL}(t)$. If players apply so called Markov feedback strategies, they react indirectly to each other's past decisions, as far as these are reflected in the current value of the state variables (K^1 and K^2). Therefore, each firm must take into account how its decisions will influence the state of the system and hence future decisions of the competitor. Feedback investment strategies will be functions of the capital stocks, $I^{iFB}(K^1, K^2)$.

It is not clear ex ante how firms will react to a higher competing capital stock, i.e. whether the derivative $\frac{\partial I^{iFB}}{\partial K^j}$ is positive or negative. The effect of the competitor's capital stock on investment can be negative for the reason that a higher capital stock of the competitor implies that the competitor produces more output. This decreases the profitability of output and investment to the home firm. The effect can be positive, though, for the reason that a higher competing capital stock induces the firm to increase its investments to keep its market share.

The equilibrium under environmental taxes and standards with open-loop investment strategies was analyzed in Feenstra, Kort, Verheyen and De Zeeuw (1996). To compute an equilibrium of feedback strategies for the general formulation of the differential game in section 2 is difficult. But it is possible to approximate the steady-state capital stock in the feedback equilibrium for explicit functional forms. Therefore consider the following scenario:

Prices of output follow from market equilibrium on a world market with a linear inverse demand curve, $p = p_0 - x^i - ax^j$. The parameter a, $0 \leq a \leq 1$, denotes the degree of substitutability between the products. Gross revenues, R^i, are given by $R^i = px^i = p_0x^i - (x^i)^2 - ax^ix^j$.

Technology is characterized by the Cobb-Douglas production function $x^i = \sqrt{e^i}K^{i\beta}$.

Investment costs are quadratic, $C(I^i) = \frac{1}{2}c(I^i)^2$.

Furthermore, it is assumed that the two countries are symmetric, that is, they are assumed to have equal rates of depreciation and discount, d and r, equal production functions and the same targets of environmental policy, $\overline{e}$. The technology is increasing returns to scale, so that $\beta > \frac{1}{2}$. Marginal productivity of capital is decreasing, so that $\beta < 1$. It is straightforward to check that these functional forms satisfy the assumptions made in section 2.

With these functional forms the expression for the Nash equilibrium value of energy use in section 3.1 in case of taxes in both countries becomes:

$$e^{iN}(K^i, K^j) = \frac{p_0^2 K^{i\,2\beta}(2(p^e + \tau^j) + (2-a)K^{j\,2\beta})^2}{(4(p^e + \tau^i + K^{i\,2\beta})(p^e + \tau^j + K^{j\,2\beta}) - a^2 K^{i\,2\beta} K^{j\,2\beta})^2}. \tag{16}$$

Insert the functions in the differential game to obtain:

$$\max_{I^i} \int_0^\infty e^{-rt}\Pi^i(K^i, K^j)dt \tag{17}$$

$$\text{s.t. } \dot{K}^i = I^i - dK^i \tag{18}$$

$$\dot{K}^j = I^j - dK^j \tag{19}$$

$$I^i \geq 0 \tag{20}$$

where

$$\Pi^i(K^i, K^j) = \frac{p_0^2 K^{i\,2\beta}(2(p^e + \tau^j) + (2-a)K^{j\,2\beta})^2(p^e + \tau^i + K^{i\,2\beta})}{(4(p^e + \tau^i + K^{i\,2\beta})(p^e + \tau^j + K^{j\,2\beta}) - a^2 K^{i\,2\beta} K^{j\,2\beta})^2} - \frac{1}{2}cI^{i\,2}dt. \tag{21}$$

In case of standards in both countries, $e^i = \overline{e}^i$ and

$$\Pi^i(K^i, K^j) = p_0\sqrt{\overline{e}^i}{K^i}^\beta - \overline{e}^i {K^i}^{2\beta} - a\sqrt{\overline{e}^i}\sqrt{\overline{e}^j}{K^i}^\beta {K^j}^\beta - p^e\overline{e}^i - \frac{1}{2}c{I^i}^2 dt. \tag{22}$$

The derivative of marginal profits to foreign capital, Π^i_{ij}, is negative for both policy instruments and for all values of capital and energy use. The derivative of marginal profits to own capital, Π^i_{ii}, is assumed to be negative. For standards, this requires p_0 to be large enough, for taxes it requires an upper limit on $(p_e + \tau)$. It can be derived that the second-order derivative of profits to foreign capital, Π^i_{jj}, is always positive under standards. Under taxes, $\Pi^i_{jj} > 0$ requires again that $(p^e + \tau)$ is not too large. In the sequel it is assumed that this is the case.

These objective functions are approximated with revenue functions that are linear-quadratic in (K^i, K^j). This gives an approximation to the feedback steady-state solution. The approximation is found by application of the following algorithm:

step 1) Choose a starting point, (K_0, K_0).

step 2) Compute a second-order Taylor approximation of the objective function in the neighbourhood of this starting point.

step 3) Determine the steady-state capital stocks for this approximation analytically.

step 4) Take the resulting steady state as the new starting point and return to step 2. Repeat the algorithm until the new steady state is close enough to the old one.

When convergence occurs, this is at a point (K^*, K^*), which is the steady-state capital stock for the Taylor approximation of the objective function around the same point, (K^*, K^*). In appendix A, step 2 and 3 of this algorithm are elaborated.

With the above algorithm, we can compute equilibrium linear investment strategies, $I(K^i, K^j) = P_1K^i + P_3K^j + P_4$, and steady-state capital stocks, $\overline{K}$, under taxes and standards, for a target of environmental policy, $\overline{e}$. These strategies are approximations to the feedback equilibrium strategies in a small neighbourhood of (K^*, K^*).

The sign of P_3 is interesting, because it determines the type of strategic interaction in case of feedback investment strategies. If $P_3 > 0$ a firm reacts to a higher capital stock of its competitor with higher investments. If $P_3 < 0$ the reverse is the case. The following proposition shows that the only possibly stable solution has a negative P_3 under a condition on the second order derivatives of (21), respectively (22). Furthermore a sufficient condition for this solution to be stable is given. The conditions can be shown to hold for reasonable values of the parameters both for taxes and for standards. The proof of the proposition is given in appendix B.

Proposition 1

For

$$\Pi^i_{jj} \leq \frac{1}{4}(2d+r)^2c + 2\Pi^i_{ij} - \Pi^i_{ii} \tag{23}$$

one or more solutions exist. Only one solution can be stable and for this solution $P_3 < 0$. *If, in addition,*

$$\Pi^i_{ij} < min[-\frac{1}{2}r^2c, -\frac{1}{4}\sqrt{6}], \tag{24}$$

then this solution is indeed stable.

In other words, under the conditions of the proposition, a unique stable feedback equilibrium steady-state capital stock, (K^*, K^*), exists for the approximated objective function and the corresponding equilibrium investment strategy has $P_3 < 0$. Under taxes, for all β, both conditions in the proposition are satisfied, given the parameter values used. Under standards, the conditions in the proposition are satisfied for most parameter values. However, it may happen that $\Pi^i_{jj} > \frac{1}{4}(2d+r)^2c + 2\Pi^i_{ij} - \Pi^i_{ii}$. In that case, multiple stable equilibria may exist and there is a coordination problem on the choice of equilibrium strategies. For β and $\bar{e}$ low enough, however, one can exclude this case (see appendix C). Therefore, for both taxes and standards the effect of foreign capital on own investments, $\frac{\partial I^{i\,FB}}{\partial K^j} = P_3$, is negative around the steady state, for reasonable parameter values.

Given the decreasing marginal productivity of capital it is to be expected that the derivative of investment with respect to own capital, $\frac{\partial I^{i\,FB}}{\partial K^i} = P_1$, is negative. This is indeed true for realistic values of d, r and c, which is formally derived in appendix D. For -unreasonably- large values of the parameters mentioned, it is possible to find a positive P_1, due to indirect effects of capital on equilibrium emissions.

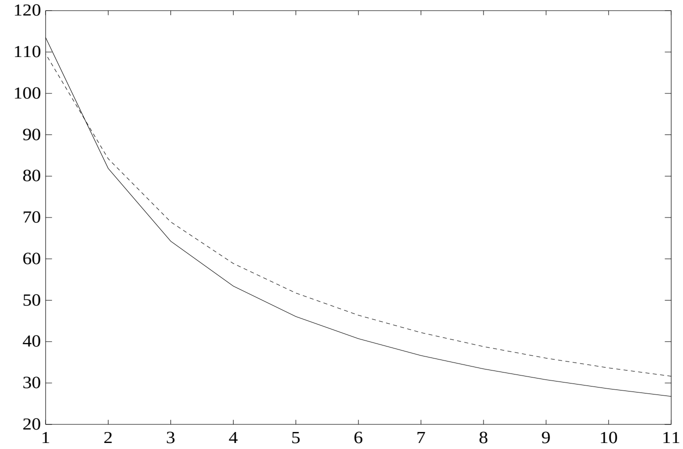
Taxes and standards are to be compared in the feedback equilibrium steady state. Figure 1 shows steady-state capital stocks and firm profits for both policy instruments as a function of the environmental target. Before-tax profits are also shown, to enable a good comparison of taxes and standards. Of course, after-tax profits are always substantially lowered by the tax-payment. Since compensation is possible, it is better to compare before-tax profits with profits under standards.

figure 1: Capital stocks and profits as a function of the environmental target.

Parametervalues: β =0.7, c=2, d=0.10, r=0.08, p^e =1, a=1.

Capital stocks:

K ↑

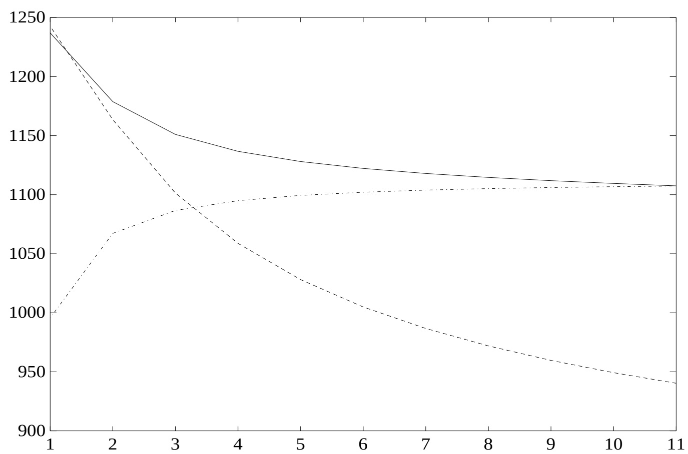


e →

- - - - : standards
____ : taxes

Profits:

Π ↑



e →

- - - - : standards
____ : before-tax profits
- . - . : after-tax profits

Remember that in this model, due to rivalry between firms, investment is too high, so that profits are lower, the higher the capital stock. It is clearly not true that taxes always result in higher capital stocks than standards. On the contrary, for most parameter values, standards result in higher capital stocks. That is due to the investments that are carried out for strategic reasons. As a consequence profits are larger under taxes for most parameter values, even when after-tax profits are considered. The value of β is an important parameter in this respect, as well as the stringency of environmental policy. The laxer environmental policy, the more likely it is that standards result in higher steady-state capital stocks and lower profits than taxes[2].

3.3 Strategic effects

To explain the effect of strategic behaviour on investment under environmental policy, we use so called Net Present Value expressions. These expressions can be derived from forward integration of first-order conditions. They give the properly discounted future stream of extra profits due to an additional unit of capital at time t (See e.g., Hartl and Kort, 1996).

First, consider the cooperative solution. From the point of view of the two firms as a cartel, this is the optimal outcome. It is found as the solution to the following optimal-control model:

$$\max_{I^i, I^j} \int_0^\infty e^{-rt}[\Pi^i + \Pi^j]dt \tag{25}$$

$$\dot{K}^i = I^i - dK^i, \quad \text{for } i = 1,2 \tag{26}$$

with $e^i = e^{iC}$. From the first-order conditions the following Net Present Value expression can be derived. Optimal investment is characterized by:

$$\int_t^\infty e^{-(r+d)(s-t)}[\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i} + \frac{\partial R^j}{\partial x^i}\frac{\partial x^i}{\partial K^i}]ds - C'(I^i(t)) = 0 \tag{27}$$

The term in brackets gives the extra revenues at time s due to an additional unit of capital stock, invested at time t. Future revenues are discounted with rate r and corrected for depreciation at rate d, since one unit of capital bought at time t reduces in value to $e^{(r-d)(s-t)}$ units at time s. To obtain this unit, the firm must spend $C'(I^i))$ at time t. In case

[2]Provided that a unique stable equilibrium exists.

of cooperation, the -negative- effect of increased home capital on earnings of the foreign firm is taken into account. All strategic interaction is absent, since the firms cooperate.

Second, consider equilibria where the firms compete in a Cournot-Nash fashion. The Net Present Value expression for firms that choose investments according to an open-loop strategy was derived in Feenstra, Kort, Verheyen and De Zeeuw (1996):

$$\int_t^\infty e^{-(r+d)(s-t)}[\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i} + \frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^j}{\partial K^i}]ds - C'(I^i(t)) = 0 \tag{28}$$

This expression contains an additional term, $\frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^j}{\partial K^i}$, due to strategic interaction. Firms try to influence energy and, hence, output decisions of their competitor with their capital stock. If they manage to decrease foreign output, their own revenues increase ($\frac{\partial R^i}{\partial x^j} < 0$). Since $\frac{\partial x^j}{\partial e^j}\frac{\partial e^j}{\partial K^i} < 0$, when taxes are applied as an instrument of environmental policy, this strategic effect leads to more investment. In case of environmental standards, however, $\frac{\partial e^j}{\partial K^i} = 0$ because firms are on the boundary of a binding constraint. In that case the strategic effect is absent and investments are lower.

Finally consider the Net Present Value expression when firms use feedback investment strategies. This requires first-order conditions for an equilibrium, which can for instance be found in Feichtinger and Hartl (1986, p536). In our model this leads to:

$$C'(I^i) = \mu^{ii} \tag{29}$$

$$\dot{\mu}^{ii} = (r+d)\mu^{ii} - [\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i} + \frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^j}{\partial K^i} + \mu^{ij}\frac{\partial I^j}{\partial K^i}] \tag{30}$$

$$\dot{\mu}^{ij} = (r+d)\mu^{ij} - \mu^{ij}\frac{\partial I^j}{\partial K^j} - [\frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial K^j} + \frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^j}{\partial K^j}] \tag{31}$$

Here μ^{ii} denotes the shadow value of own capital to firm i. Its value is determined by condition (30). This includes the term $\mu^{ij}\frac{\partial I^j}{\partial K^i}$ because of feedback reactions. The variable μ^{ij} denotes the shadow value of foreign capital to firm i. Condition (31) determines its value. Note that these first-order conditions are not sufficient to actually compute candidate solutions, since in general one does not know $\frac{\partial I^i}{\partial K^j}$ and $\frac{\partial I^j}{\partial K^j}$. Integrate forward and rearrange, to find the following Net Present Value expression:

$$\int_t^\infty e^{-(r+d)(s-t)}[\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i} + \frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^j}{\partial K^i} + \mu^{ij}\frac{\partial I^j}{\partial K^i}]ds - C'(I^i(t)) = 0 \tag{32}$$

with μ^{ij} given by

$$\mu^{ij}(s) = \int_s^\infty e^{-(r+d-\frac{\partial I^j}{\partial K^j})(\tau-s)}[\frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial K^j} + \frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^j}{\partial K^j}]d\tau \tag{33}$$

In case of standards in both countries, both firms are at a corner solution for energy use. Therefore, these formulas then become:

$$\int_t^\infty e^{-(r+d)(s-t)}[\frac{\partial R^i}{\partial x^i}\frac{\partial x^i}{\partial K^i} + \mu^{ij}\frac{\partial I^j}{\partial K^i}]ds - C'(I^i(t)) = 0 \tag{34}$$

with μ^{ij} given by

$$\mu^{ij}(s) = \int_s^\infty e^{-(r+d-\frac{\partial I^j}{\partial K^j})(\tau-s)}[\frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial K^j}]d\tau. \tag{35}$$

Compared to (28), an additional strategic effect exists that works through investment, $\mu^{ij}\frac{\partial I^j}{\partial K^i}$. This effect can be strong enough to outweigh the first strategic effect. From a comparison of the net present value expressions follows immediately:

Proposition 2:
When $\frac{\partial R^i}{\partial x^j}\frac{\partial x^j}{\partial e^j}\frac{\partial e^j}{\partial K^i}^N + \mu^{ij}(tax)\frac{\partial I^j}{\partial K^i} < \mu^{ij}(sta)\frac{\partial I^j}{\partial K^i}$ for all t, the strategic incentive for investment is greater under standards than under taxes.

The term $\mu^{ij}\frac{\partial I^j}{\partial K^i}$ captures the indirect strategic effect. If firm i increases its investment, its capital stock grows. This influences the investment by firm j immediately as expressed by $\frac{\partial I^j}{\partial K^i}$. The shadow price μ^{ij} denotes the valuation by firm i of such a change in firm j's investment. Thus μ^{ij} gives the valuation by firm i of an additional amount of capital owned by firm j. This is given by the properly discounted flow of marginal decreases in firm i's revenues if firm j owns an extra amount of capital (cf. (33) and (35)). Since firm i's revenues decrease when competition from firm j increases, μ^{ij} will have a negative sign. If the derivative of I^j to K^i is negative, which we have shown to be the case for reasonable parameter values (cf. proposition 1), then the additional strategic effect is positive. This effect, $\mu^{ij}\frac{\partial I^j}{\partial K^i}$, can be higher for standards than for taxes, since firms are less flexible in case of binding standards and do not substitute energy for capital. To explain this, first note that with feedback investment strategies, given that $\frac{\partial I^j}{\partial K^i} < 0$, a marginal increase in capital K^i leads firm j to invest less and hence to decrease its capital stock, K^j. This in turn leads firm j to decrease its output. It depends on the sign of $\frac{\partial e^j}{\partial K^j}$ whether j's use of energy also decreases. If substitution effects dominate, e^j increases. In case of standards, substitution between production factors - i.e. an increase in e^j in reaction to a decrease in K^j - will not occur, since the standard is an upper bound to e^j. A decrease in e^j will not happen since the firm is at a corner solution for its energy use. Hence, $\frac{\partial e^j}{\partial K^j}$ equals zero. In case of taxes, on the contrary, the firm is flexible to adjust its use of energy in an optimal way to the capital stocks. Substitution between energy and capital then partly cancels the effect on output of an increased capital stock. That implies that the change in firm j's output and hence the relevant strategic effect, $\mu^{ij}\frac{\partial I^j}{\partial K^i}$, can be greater with standards than with taxes, provided substitution effects dominate marginal-productivity effects.

The proposition contradicts earlier conclusions by Ulph (1992). He derived that standards always result in less strategic investments than taxes. Ulph used a multistage static model. That implied that his subgame-perfect equilibrium is equivalent to an equilibrium with feedback strategies for energy use, but open-loop strategies for investment, as represented by the Net Present Value expression (28). His model does not allow for strategic interaction between firms in investment strategies. Only then the result holds that taxes always provide larger incentives for investment than standards.

To conclude, it is not generally true that standards moderate strategic overinvestment when firms react on each others behaviour by adjustments in their investment plans. Although standards commit firms to a certain use of energy input, they may drive firms to more investment than taxes. In that case the use of standards as a commitment device does not work. Then, the use of taxes as an environmental policy instrument is to be preferred when, next to environmental targets, profits of domestic firms are an important objective to the government.

4 Comparative statics for parameter values

Figure 1 shows that standards may result in more strategic investment than taxes. As a consequence, firm profits are higher under taxes than under standards. For some strict environmental policy targets the reverse is true, however. This section discusses the values chosen for the parameters that may influence this result. We show some comparative static results that indicate how the difference between taxes and standards changes with parameter values. The relationship between the parameters of the model and the steady-state values of the capital stock in the feedback solution is complex. It involves the approximation algorithm and the computation of the roots of a third degree polynomial in the third step of this approximation algorithm. Therefore we can not give simple expressions that link parameter values to the conditions in the propositions. But numerical experimentation gives some clues about the direction of changes.

First consider the parameter β in the production function. The literature on econometric production functions provides empirical estimates of returns to scale in large energy-intensive industries. These gave a range around 1.2 (Morrison, 1993, 1994, Ilmakunnas and Törmä, 1994, Pindyck and Rotemberg, 1983). To satisfy the model assumptions, $\frac{1}{2} < \beta < 1$ must hold. Given that the Cobb-Douglas coefficient of energy equals $\frac{1}{2}$, returns to scale then vary between 1 and 1.5. Compared to 1.2, this seems reasonable given the empirical literature.

Now consider the price of energy, p^e. When prices are extremely high, they provide enough incentives for firms to reduce energy use and environmental policy is superfluous. Therefore, we took the relatively low value 1 for this price.

figure 2: Emissions for no policy case, as a function of β.
Parametervalues: c=2, d=0.10, r=0.08, p^e =1, a=1.

Emissions:

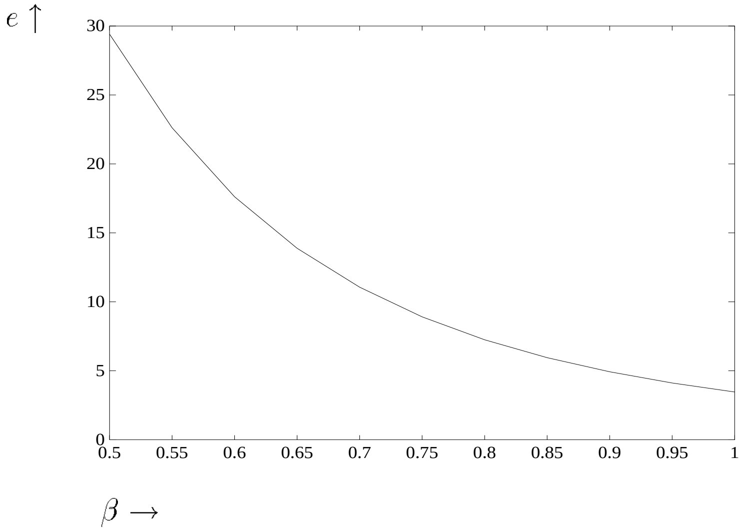


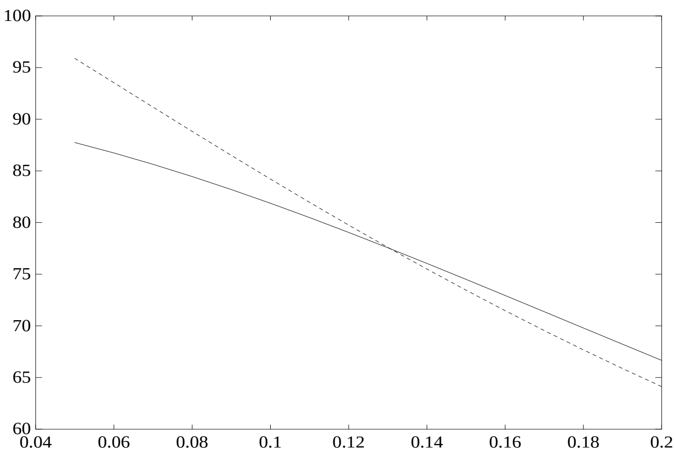
Figure 2 depicts emissions, when no environmental policy is applied and p^e has the value 1. It shows that for values of β close to 1, even with this low value for p^e, the range of meaningful emission goals is limited. When β is high, energy is a relatively unimportant production factor, and for lax environmental policy goals, environmental policy is unnecessary. The higher the price of energy, p^e, the more important this effect is.

For d, the rate of depreciation, we took the value 0.10. For r, the rate of discount we took the value 0.08. Figure 3 shows the effect of different values for d. If d is higher, capital depreciates faster. That implies that commitment with regard to future output provided by an additional unit of capital decreases. The consequence is a lower steady-state capital stock and a smaller difference between standards and taxes. For d high enough, the difference changes sign, because strategic interaction through commitment on energy use dominates strategic interaction through commitment on investments. Then taxes result in more strategic investment than standards. Increases in r have a similar effect. A higher rate of discount implies that the future is less important relative to the present. Current costs of investment then become important relative to future earnings. The steady-state capital stock hence decreases. The dynamic aspects that cause a difference between environmental policy instruments in strategic interaction through commitment on investment loose importance. As a result, the difference between standards and taxes decreases and eventually changes sign, when the static aspect of strategic interaction through commitment on energy use dominates.

figure 3: Effect on capital stock and profits of changes in d.
Parametervalues: $\beta = 0.7$, c=2, r=0.08, $p^e = 1$, a=1.

Capital stocks:

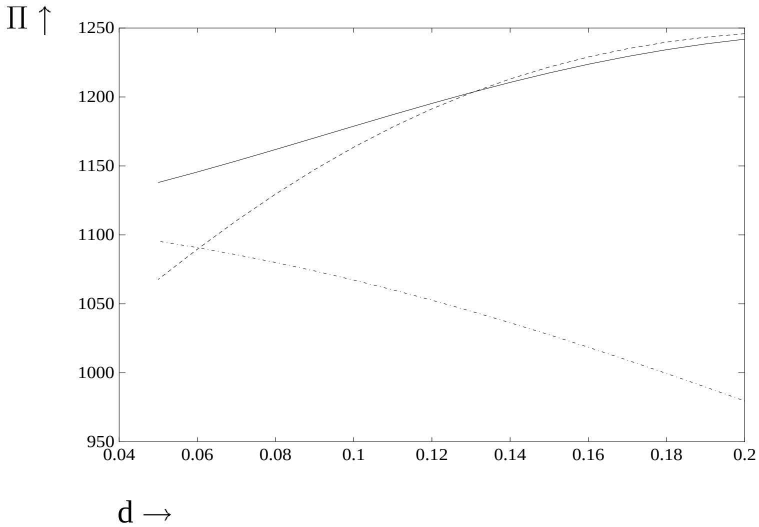
K ↑



d →

- - - - : standards
____ : taxes

Profits:



- - - - : standards
____ : before-tax profits
- . - . : after-tax profits

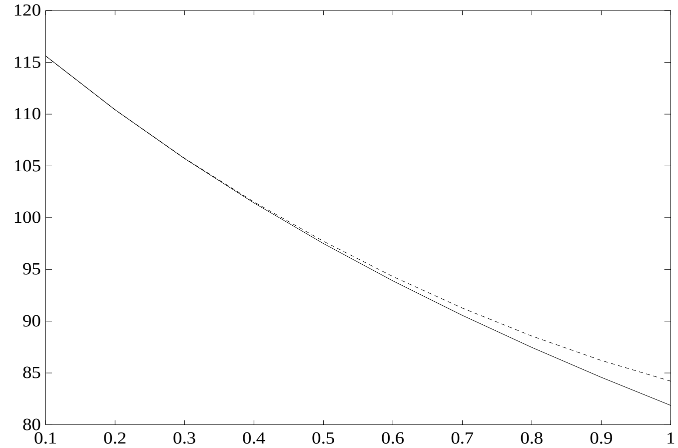
For c, that is a parameter for adjustment costs of investment, we took the value 2. The higher c, the higher the costs of investment, and the lower therefore the steady-state stock of capital. Changes in capital stock become more expensive, so that commitment through investment is less attractive. Therefore the differences between standards and taxes decrease with c.

The parameter a, finally, denotes the interconnectedness of markets. If $a = 0$, firms do not influence each others prices. In that case both firms are monopolists and strategic effects disappear. Environmental taxes and standards then have equal effects on firms in the model above. The higher a, the more effect firms have on each others prices and the more important strategic effects become. For $a = 1$ the outputs of the two firms are perfect substitutes. Figure 4 shows equilibrium capital stocks and profit rates for different values of a. The difference between standards and taxes increases with a, because strategic effects gain in importance. For that reason we took a to be equal to 1 in the other figures.

figure 4: Effect on capital stock and profits of changes in a.
Parametervalues: $\beta = 0.7$, c=2, d=0.10, r=0.08, $p^e = 1$.

Capital stocks:

K ↑

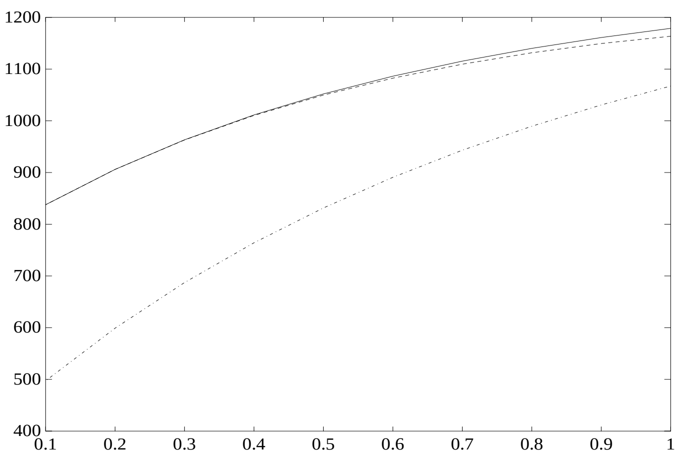


a →

- - - - **:** standards
____ **:** taxes

Profits:

Π ↑



a →

- - - - **:** standards
____ **:** before-tax profits
- . - . **:** after-tax profits

The results in this section show again that it is not true that taxes always result in more investment than standards for strategic reasons, and therefore lower profit rates. The reverse, that standards lead to more investment and lower profits is neither valid. Which environmental policy instrument is 'better', which means here that it results in the highest profits for a given emission target, depends on the precise values of the parameters.

5 Conclusions

This paper is concerned with international rivalry and environmental policy. In a multistage static framework Ulph (1992) found that environmental taxes lead to higher investment than standards. Firms that compete on an international market have an incentive to increase investments to gain strategic advantage. But in equilibrium, competitors act similar so that higher capital stocks, more output and lower profits result. Therefore, ceteris paribus, governments prefer standards rather than taxes, due to the reduction in strategic investment. This was confirmed in Feenstra, Kort, Verheyen and De Zeeuw (1996), in a differential-game framework with open-loop investment strategies. The differences between environmental taxes and standards as regards their effect on strategic behaviour are due to their influence on the flexibility of firms. Standards reduce a firm's flexibility in its choice of emissions, while taxes do not.

In this paper, flexibility in investment behaviour is introduced. More specifically, we consider feedback investment strategies. Under feedback investment strategies, firms have to take into account that the competitor will react through its investment on marginal increases in the capital stock. The effects of environmental standards and taxes on this type of strategic behaviour differ. It turns out that it cannot -as in the case of open-loop investment strategies- be stated that taxes will always lead to more investments than standards. It is shown in the paper that if substitution effects between production factors are large enough, investment is larger under standards than under taxes. Ceteris paribus, governments then prefer taxes rather than standards as environmental policy instrument.

Note that in the current model, consumer surpluses are neglected. It is implicitly assumed that firms sell a substantial part of their output to third countries. Ulph (1996) and Kennedy (1994) include consumer surplus in a multistage static game. Since it is in the interest of consumers that competition between the two firms results in more output, they favour larger investments. Inclusion of consumer surplus would therefore require a different valuation of high capital stocks. When consumer surplus is important, taxes may be preferred by the government over standards, while still taxes lead to more investment than standards.

A differential game model of a duopoly describes the behaviour of firms that compete in an international market. After equilibrium behaviour on energy choice has been inserted, a capital accumulation game results. The two firms choose investment rates, to maximize their objectives, given government policy and the strategy of the competitor. Feedback equilibria in capital accumulation games are derived in Reynolds (1987) and in Fershtman and de Zeeuw (1992). The game in this paper is a bit different, because investment has indirect effects on output through energy use. That implies that the objective function is not linear quadratic in capital stocks like in the two papers mentioned above. But it is possible with the help of an approximation algorithm to extend the method employed in Fershtman and de Zeeuw and find approximations to the steady state.

The indirect effects through energy choices and the approximation step imply that equilibrium investment depends in a complex way on domestic and foreign capital stocks. In particular, the sign of the derivative of equilibrium investment to foreign capital may be either negative or positive. Conditions are given for stable equilibria that are characterized by a negative sign of this derivative. Although these conditions capture the cases that are economically most relevant, an exceptional case with a positive derivative is still possible.

Flaherty (1980) analyzes a capital accumulation game with symmetric and asymmetric equilibria. With the help of linear approximations, it is shown that the asymmetric equilibria are stable and the symmetric equilibrium is not. Although in a different context, this shows that stable asymmetric equilibria are certainly an option in this type of models. For tractability reasons, our analysis has been restricted to the case of symmetry, where both countries apply the same type of environmental policy. Extension to the asymmetric case where one country applies taxes and the other standards is an option. However, we do not expect that a (complex) analysis of the asymmetric case would change our conclusion with respect to strategic behaviour.

A Details of the algorithm

In this appendix details are given about the 2^{nd} and 3^{rd} step of the algorithm in the main text.

Step 2: A second order Taylor approximation of the objective function of the differential game results in a profit function:

$$\Pi^i = \frac{1}{2}K^tQK \ + \ q^tK \ + \ q_0 - \frac{1}{2}cI_i^2 \tag{A.1}$$

where Q is a symmetric matrix $\begin{bmatrix} Q_1 & Q_3 \\ Q_3 & Q_2 \end{bmatrix}$, q is a column vector (q_1, q_2), q_0 is a scalar and K is a column vector (K^1, K^2). The Q_i denote the second derivatives of profits to capital, respectively $Q_1 = \Pi^i_{ii}$, $Q_2 = \Pi^i_{jj}$ and $Q_3 = \Pi^i_{ij}$. Furthermore,

$$q_1 = \Pi^i_i - K_0Q_1 - K_0Q_3$$

$$q_2 = \Pi^i_j - K_0Q_3 - K_0Q_2.$$

Step 3:
For a linear-quadratic objective function, the analytical expression of the steady state can be computed. A model with $Q_2 = 0$, $Q_1 = -2$, $Q_3 = -1$, $q_2 = 0$ and $q_1 = a > 0$, has been solved by Reynolds (1987) and by Fershtman and De Zeeuw (1992). Below, the approach of the latter paper is followed to obtain a solution to the approximated game.

Consider the differential game with Π^i given by (A.1) as the objective function. This is a game with a quadratic objective function, linear dynamics and two state variables. The dynamic-programming approach is used to find a linear Markov-perfect equilibrium. The Hamilton-Jacobi-Bellman equations for the game are:

$$rV^i(K^i, K^j) =$$

$$\max_{I^i}\{-\frac{1}{2}c(I^i)^2 + \frac{1}{2}K^tQK + q^tK + q_0 + \frac{\partial V^i}{\partial K^i}(I^i - dK^i) + \frac{\partial V^i}{\partial K^j}(I^j - dK^j)\} \tag{A.2}$$

The optimal I^i is given by $I^i = \frac{1}{c}\frac{\partial V^i}{\partial K^i}$. The quadratic value function has the form:

$$V^i = \frac{1}{2}K^tPK + p^tK + p_0 \tag{A.3}$$

with P a symmetric matrix $\begin{bmatrix} P_1 & P_3 \\ P_3 & P_2 \end{bmatrix}$, p a columm-vector (p_4, p_5) and p_0 a scalar. The partial derivatives of the value function become:

$$\frac{\partial V^i}{\partial K^i} = P_1 K^i + P_3 K^j + p_4 \tag{A.4}$$

and

$$\frac{\partial V^i}{\partial K^j} = P_3 K^i + P_2 K^j + p_5 \tag{A.5}$$

Hence the optimal investment strategy can be written:

$$I^i = \frac{1}{c}(P_1 K^i + P_3 K^j + p_4) \tag{A.6}$$

Because of symmetry, it suffices to consider the Hamilton-Jacobi-Bellman equation of only one firm. For the other firm the computations are analogous. Insert the optimal solution for investment, given by (A.6), and the value function (A.3), with derivatives (A.4) and (A.5), into equation (A.2) and note that it must hold for each K. Equations for the elements of P and p result if the coefficients of the quadratic expression in K are equated to zero:

$$P_1^2 + 2P_3^2 - (2d + r)cP_1 + Q_1 c = 0 \tag{A.7}$$

$$P_3^2 + 2P_1 P_2 - (2d + r)cP_2 + Q_2 c = 0 \tag{A.8}$$

$$2P_3 P_1 + P_3 P_2 - (2d + r)cP_3 + Q_3 c = 0 \tag{A.9}$$

$$P_1 p_4 + P_3 p_4 + P_3 p_5 - c(d + r)p_4 + q_1 c = 0 \tag{A.10}$$

$$P_3 p_4 + P_2 p_4 + P_1 p_5 - c(d + r)p_5 + q_2 c = 0 \tag{A.11}$$

$$p_4^2 + 2p_4 p_5 - 2cq_0 - 2crp_0 = 0 \tag{A.12}$$

If the investments strategies (A.6) must also result in a stable capital growth path, two stability conditions follow from solving for (18) and (19) with (A.6):

$$P_1 + P_3 - dc < 0 \tag{A.13}$$

$$P_1 - P_3 - dc < 0 \tag{A.14}$$

The first three equations, (A.7) to (A.9), form an independent system of equations in P_1, P_2 and P_3. These equations define conic sections in the $(P1, P3)$-plane, respectively an ellipse, a parabola and a hyperbola. This suggests to apply polar coordinates. First note that the ellipse has its centre at $(\frac{1}{2}(2d+r)c, 0)$, the parabola its top at $(\frac{1}{2}(2d+r)c-\frac{Q_2 c}{2P_2}, 0)$ and the hyperbola its centre at $(\frac{1}{2}(2d+r)c-\frac{1}{2}P_2, 0)$. With the transformation $p_1 = P_1-\frac{1}{2}(2d+r)c$ and $p_3 = \sqrt{2}P_3$ the three equations simplify to:

$$\frac{p_1^2}{\rho^2} + \frac{p_3^2}{\rho^2} = 1 \tag{A.15}$$

$$\frac{1}{2}p_3^2 + 2P_2 p_1 + Q_2 c = 0 \tag{A.16}$$

$$\sqrt{2}p_3 p_1 + \frac{1}{2}\sqrt{2}p_3 P_2 + Q_3 c = 0 \tag{A.17}$$

with

$$\rho^2 = \frac{1}{4}(2d+r)^2 c^2 - Q_1 c \tag{A.18}$$

Equation (A.15) defines a circle with radius ρ and centre at the origin. The polar coordinates R and ϕ are introduced in the (p_1, p_3) plane (with origin $(\frac{1}{2}(2d+r)c, 0)$ in the (P_1, P_3) plane):

$$p_1 = R\sin\phi \tag{A.19}$$

$$p_3 = R\cos\phi \tag{A.20}$$

so that in the original variables:

$$P_1 = \frac{1}{2}(2d+r)c + R\sin\phi \tag{A.21}$$

$$P_3 = \frac{1}{2}\sqrt{2}R\cos\phi \tag{A.22}$$

with $-\pi < \phi < \pi$. The P_2 axis is left unchanged.

From equation (A.15) it follows that

$$R^2 = \rho^2. \tag{A.23}$$

Rewrite equation (A.16) to an expression for P_2, do the same with equation (A.17) and equate these two:

$$\frac{-(2Q_2 c + p_3^2)}{4p_1} = \frac{-(\sqrt{2}Q_3 c + 2p_1 p_3)}{p_3}. \tag{A.24}$$

Now insert (A.23), (A.19) and (A.20) in (A.24). Divide by $\cos\phi$ and realize that $\frac{\sin\phi}{\cos\phi} = \tan\phi$ and that $\cos^2\phi = \frac{1}{\tan^2\phi+1}$. After multiplication with $(1+\tan^2\phi)$ and division by $-4\sqrt{2}Q_3c$ the following equation results:

$$\tan^3\phi + (\frac{-8\rho^2+2Q_2c}{-4\sqrt{2}Q_3c})\tan^2\phi + \tan\phi + (\frac{2Q_2c+\rho^2}{-4\sqrt{2}Q_3c}) = 0 \tag{A.25}$$

With the notation $y = \tan\phi$, $u = \frac{-\sqrt{2}\rho^2}{Q_3c}$, $v = \frac{-Q_2}{Q_3}\frac{1}{4}\sqrt{2}$, this can be rewritten as:

$$y^3 + (-u+v)y^2 + y + (\frac{1}{8}u+v) = 0 \tag{A.26}$$

This is a third degree polynomial. Its roots, y_1, y_2 and y_3 might be real or complex, dependent on the values of u and v. The three roots are given by the formulas:

$$y_1(u,v) = \frac{1}{3}(u-v) + 2\sqrt{-q}\cos(\frac{\psi}{3}) \tag{A.27}$$

$$y_2(u,v) = \frac{1}{3}(u-v) + 2\sqrt{-q}\cos(\frac{\psi}{3}+\frac{2\pi}{3}) \tag{A.28}$$

$$y_3(u,v) = \frac{1}{3}(u-v) + 2\sqrt{-q}\cos(\frac{\psi}{3}-\frac{2\pi}{3}) \tag{A.29}$$

$$q = \frac{1}{3} - \frac{1}{9}u^2 + \frac{1}{9}(2uv - v^2) \tag{A.30}$$

$$r = \frac{u}{3}(\frac{u^2}{9} - \frac{11}{16}) + \frac{v}{3}(\frac{uv}{3} - \frac{u^2}{3} - \frac{v^2}{9} - 1) \tag{A.31}$$

where ψ is defined by

$$\cos\psi = \frac{r}{(-q)^{\frac{3}{2}}} \tag{A.32}$$

and

$$0 < \psi \leq \pi \tag{A.33}$$

Stable solutions for the feedback equilibrium are obtained from roots that after the appropriate transformations[3] result in P_1, P_2 and P_3 that satisfy the stability requirements (A.13) and (A.14). In polar coordinates these stability conditions read:

$$\rho[\sin\phi + \frac{1}{2}\sqrt{2}\cos\phi] < -\frac{1}{2}rc \tag{A.34}$$

[3]take $\phi = \arctan y_i$ and compute P_1, P_2 and P_3 with the help of (A.21) and (A.22)

and

$$\rho[\sin\phi - \frac{1}{2}\sqrt{2}\cos\phi] < -\frac{1}{2}rc. \tag{A.35}$$

These can only be satisfied simultaneously by $-\pi < \phi < 0$. Therefore, the interval $0 \leq \phi < \pi$ can be excluded. The stability conditions (A.34) and (A.35) can be rewritten to:

$$(1 - \frac{r^2c^2}{4\rho^2})\tan^2\phi + \sqrt{(2)}\tan\phi + \frac{1}{2} - \frac{r^2c^2}{4\rho^2} > 0 \tag{A.36}$$

$$(1 - \frac{r^2c^2}{4\rho^2})\tan^2\phi - \sqrt{(2)}\tan\phi + \frac{1}{2} - \frac{r^2c^2}{4\rho^2} > 0 \tag{A.37}$$

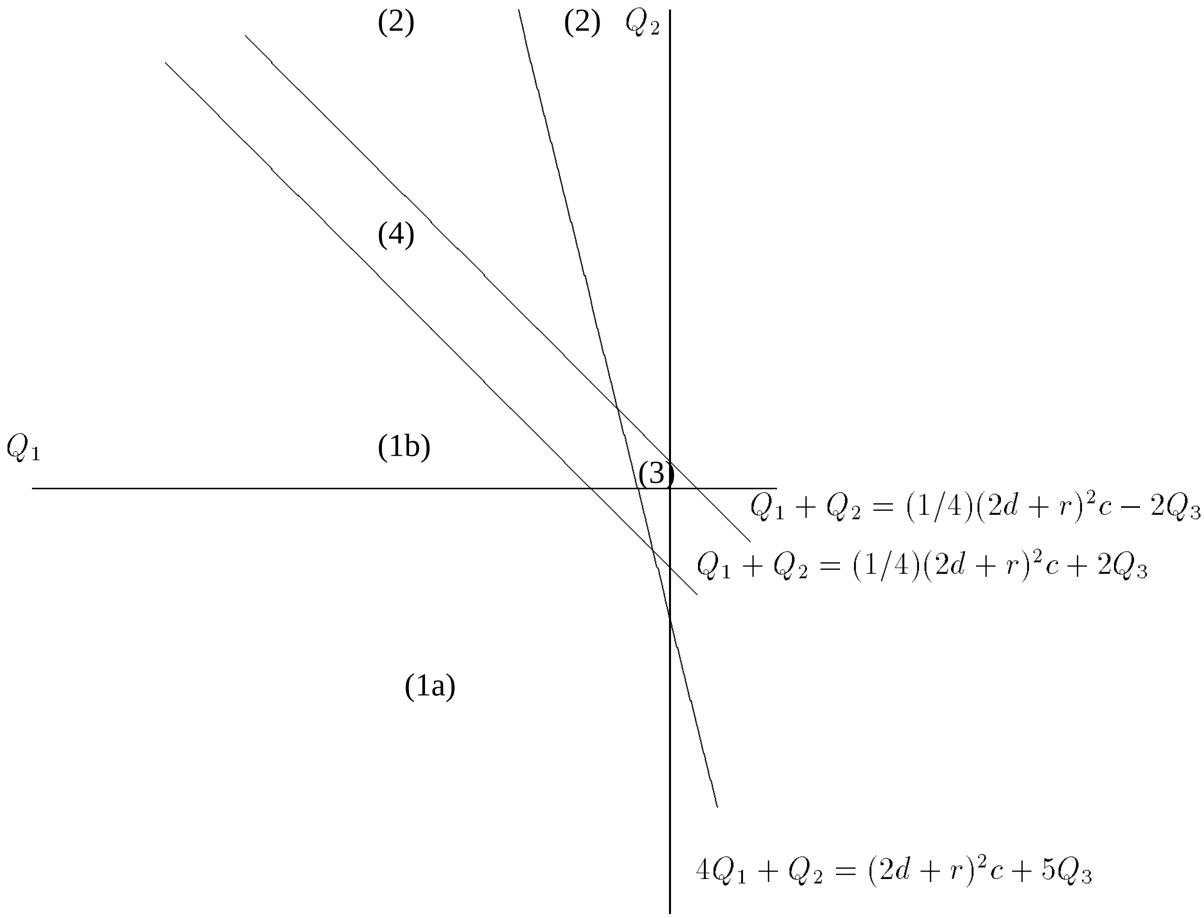
Note that $4\rho^2 - r^2c^2 > 0$ Let b_1, b_2, respectively b_3, b_4 denote the zeros of the two polynomials in $\tan\phi$. It can be proved that $b_2 > -\frac{1}{2}\sqrt{2}$ and $b_3 < \frac{1}{2}\sqrt{2}$. Also, if $|\tan\phi| < \frac{1}{2}\sqrt{2}$ then either $0 > \sin\phi \geq -\frac{1}{2}\sqrt{2}\cos\phi$ and (A.34) cannot be satisfied or $\frac{1}{2}\sqrt{2}\cos\phi \leq \sin\phi < 0$ and (A.35) cannot be satisfied. It follows that

$$\tan\phi < b_1 \text{ or } b_4 < \tan\phi \tag{A.38}$$

is a necessary condition for stability. Only $y_i = \tan\phi$ that satisfy condition (A.38) result in a stable solution.

If there is only one such a real root, a corresponding unique feedback equilibrium results. If more roots satisfy the stability requirements a situation of multiple equilibria occurs. Whether a unique root that satisfies (A.38) exists, depends on the values of Q_1, Q_2 and Q_3 in the objective function.

figure 5: Division of Q_1, Q_2-plane, for given Q_3.



In figure 5 the (Q_1, Q_2) plane is divided in 4 different regions defined by:

(1) $\quad Q_1 + Q_2 < \frac{1}{4}(2d+r)^2c + 2Q_3$

(2) $\quad Q_1 + Q_2 > \frac{1}{4}(2d+r)^2c - 2Q_3$

(3) $\quad \frac{1}{4}(2d+r)^2c + 2Q_3 < Q_1 + Q_2 < \frac{1}{4}(2d+r)^2c - 2Q_3$ and

$$4Q_1 + Q_2 < (2d+r)^2c + 5Q_3$$

(4) $\frac{1}{4}(2d+r)^2c + 2Q_3 < Q_1 + Q_2 < \frac{1}{4}(2d+r)^2c - 2Q_3$ and

$$4Q_1 + Q_2 > (2d+r)^2c + 5Q_3$$

In region (1), one real root of (A.26) that is larger than $\frac{1}{2}\sqrt{2}$ exists. Therefore, in this region there can be at most one stable solution. There is none if $y_1 < b_4$. In region (2), one real root with $y_i < -\frac{1}{2}\sqrt{2}$ exists, while other roots are not stable. Hence in this region at most one stable root exists, but none if $y_2 > b1$. In region (3) all roots satisfy $b1 \leq |y_i| \leq b_4$ and therefore no stable solution exists. In region (4), two roots with $y_i > \frac{1}{2}\sqrt{2}$ exist. When these are both larger than b_4 multiple stable equilibria exist.

For any unique root $\hat{y}$ that satisfies the stability criteria, we find $\hat{\phi} = \arctan(\hat{y})$, with $-\pi < \hat{\phi} < 0$. Together with the value for ρ from equation (A.23), this results in values for P_1 and P_3. Equation (A.8), or alternatively (A.9), gives P_2. Finally equations (A.10) to (A.12) determine p_4, p_5 and p_0. From (A.6) then follow equilibrium linear feedback investment strategies. The steady-state value of capital is found as the solution to the system of equations (18) and (19) in the steady state:

$$\overline{K}^{iFB} = \frac{-p4}{P1 + P3 - dc}. \tag{A.39}$$

The equilibrium path of capital becomes:

$$K(t) = (K_0 - \overline{K}^{FB})e^{\frac{P_1 + P_3 - dc}{c}t} + \overline{K}^{FB}. \tag{A.40}$$

We have found an analytic solution to the problem for a Taylor approximation of the original objective function around the point (K_0, K_0). The equilibrium steady-state will be the new starting point (K_0, K_0) of the algorithm. This procedure is repeated until the resulting steady state is close enough to the starting point.

B Proof of proposition 1

See figure 5 above and remember that Q_1, Q_2 and Q_3 denote Π^i_{ii}, Π^i_{jj} and Π^i_{ij} in the point (K^*, K^*). Also note that if the root of equation (A.26), y_i, is positive (negative) then, from (A.22), $P_3 = \frac{1}{2}\rho\sqrt{2}\ \cos[\arctan(y_i)]$ is negative (positive).

In appendix A it is shown that for region (1) in figure 5 at most one stable root of (A.26) exists. This region can be split up in a part with $Q_2 < 0$ (region 1a) and one with $Q_2 \geq 0$ (region 1b). From the assumptions made below equation (22) follows that only the latter region is relevant for proposition 1. It will be shown that indeed in region (1b) proposition 1 holds. That is, under the conditions in the proposition $y_1 > b_4$, so that y_1 satisfies the stability conditions and is positive. It follows that one unique stable equilibrium exists.

Define $Q_2 = \alpha(\frac{1}{4}(2d+r)^2c + 2Q_3) - \alpha Q_1$. Each point (Q_1, Q_2) in the region (1b) can be determined by a $Q_1 \leq \frac{1}{4}(2d+r)^2c + 2Q_3$ and an $\alpha \in [0, 1]$.

Look at the polynomial in y at the left-hand side of (A.26). This has a local minimum at

$$m = \frac{1}{3}\sqrt{2}\frac{1}{-Q_3}[A + \sqrt{A^2 - \frac{3}{2}}] \tag{B.1}$$

with $A = (1 - \frac{1}{4}\alpha)(\frac{1}{4}(2d+r)^2c - Q_1) - \frac{1}{2}\alpha Q_3$, given the definition of Q_2. The unique solution to (A.26) in region (1) is larger than $\frac{1}{2}\sqrt{2}$ (See appendix A). It must be the largest root of the polynomial. If the largest root is real, it must be larger than the local minimum, m. That follows from (A.27), because $\frac{1}{2} \leq cos(\frac{\psi}{3}) \leq 1$. If it can be proved that m satisfies the stability criteria, that is (see (A.38)), $b_4 < m$, then the unique solution is always stable.

The derivative of m with respect to Q_1 can be shown to be negative in the region concerned (region 1b). For fixed α, m therefore reaches its minimum value, m^*, in region (1b) for $Q_1 = \frac{1}{4}(2d+r)^2c + 2Q_3$. It is given by (B.1) with $A = -2Q_3$. Note that for the root in (B.1) to be a real number, Q_3 should satisfy $|Q_3| \geq \frac{1}{4}\sqrt{6}$.

The derivative of b_4 with respect to Q_1 can be shown to be positive in the region concerned. Hence, b_4 reaches its maximum value in (1b) for $Q_1 = \frac{1}{4}(2d+r)^2c + 2Q_3$. It is given by

$$b_4^* = \frac{1}{2}\sqrt{2} + \frac{1}{8}\sqrt{2}\frac{r^2c^2}{B} + \frac{rc}{B}\sqrt{6B + \frac{1}{2}r^2c^2} \tag{B.2}$$

with $B = -\frac{1}{4}r^2c^2 - 2Q_3$. Note that for $8Q_3 < -r^2c$, $B > 0$.

To prove that $b_4 < m$ in region (1b), it is sufficient to prove that $b_4^* < m^*$. This is equivalent to:

$$3r\sqrt{-24Q_3c - 2r^2c^2} < -8Q_3 + 16\sqrt{4Q_3^2 - \frac{3}{2}} - 4r^2c + 2\frac{r^2c}{Q_3}\sqrt{4Q_3^2 - \frac{3}{2}} \tag{B.3}$$

A sufficient condition for this to hold is that $2Q_3 < -r^2c$. This concludes the proof of proposition 1.

C Exclusion of multiple equilibria under standards

From (22) follows that in an equilibrium (K,K), $\Pi^i_{ii} = \beta(\beta-1)p_0\sqrt{\overline{e}}K^{\beta-2} - \beta(4\beta-2+a\beta-a)\overline{e}K^{2\beta-2}$, $\Pi^i_{jj} = -a\beta(\beta-1)\overline{e}K^{2\beta-2}$ and $\Pi^i_{ij} = -a\beta^2\overline{e}K^{2\beta-2}$. Add these together to see that

$$po(1-\beta) + \sqrt{\overline{e}}K^{\beta}[4\beta + a\beta - 2 - a] + a(\beta-1)\sqrt{\overline{e}}K^{\beta} - 2a\beta\sqrt{\overline{e}}K^{\beta} > 0 \qquad \text{(C.1)}$$

is a sufficient condition for

$$\Pi^i_{jj} < \frac{1}{4}(2d+r)^2 c + 2\Pi^i_{ij} - \Pi^i_{ii}. \qquad \text{(C.2)}$$

Condition (C.1) is satisfied if $4\beta > 2(1+a)$ or $\sqrt{\overline{e}}K^{\beta} < -p_0\frac{(1-\beta)}{4\beta-2(1+a)}$. Since $a \leq 1$, a sufficient condition for this is $\sqrt{\overline{e}}K^{\beta} < \frac{1}{4}p_0$. This requires β and $\overline{e}$ to be sufficiently small. If (C.2) holds, region (4) is excluded.

D Sign of P$_1$

Use definition A.21 and set $P_1 < 0$. One finds that this requires

$$|\tan\phi| > \frac{\frac{1}{2}(2d+r)c}{\sqrt{-Q_1 c}} \qquad \text{(D.1)}$$

given that $-\pi < \phi < 0$. If d, r and c are small enough, so that $Q_1 < -\frac{1}{2}(2d+r)^2 c$, the right-hand side of this inequality is smaller than $\frac{1}{2}\sqrt{2}$. It follows that any stable solution (which by necessity is characterized by $|\tan\phi| > \frac{1}{2}\sqrt{2}$) satisfies inequality (D.1) and hence has a negative P_1.

References

Brander, J.A. and B.J. Spencer, 1983, Strategic commitment with R & D: The symmetric case, Bell Journal of Economics 14(1), 225-235.

Flaherty, M. Thérèse, 1980, Industry structure and cost-reducing investment, Econometrica 48(5), 1187-1209.

Feenstra, T., P. Kort, P. Verheyen and A. de Zeeuw, 1996, Standards versus taxes in a dynamic duopoly model of trade, in: A. Xepapadeas, ed., Economic policy for the environment and natural resources (Edward Elgar, Cheltenham) 197-216.

Feichtinger, G. and R.F. Hartl, 1986, Optimale Kontrolle Ökonomischer Prozesse (De Gruyter, Berlin).

Fershtman, Ch. and A. de Zeeuw, 1992, Capital accumulation and entry deterrence: A clarifying note, in: G. Feichtinger, ed., Dynamic economic models and optimal control (Elsevier, Amsterdam) 281-296.

R.F. Hartl and P.M. Kort, 1996, Capital accumulation of a firm facing an emissions tax, Journal of Economics 63, 1-23.

Ilmakunnas, P. and H. Törmä, 1994, Energy crises and change of technology, Journal of Applied Econometrics 9, 305-320.

Kennedy, P.W., 1994, Equilibrium pollution taxes in open economies with imperfect competition, Journal of Environmental Economics and Management 27(1), 49-63.

Morrison, C.J., 1993, Investment in capital assets and economic performance: The US chemicals and primary-metals industries in transition, Journal of Business and Economic Statistics 11(1), 45-60.

Morrison, C.J., 1994, The cyclical nature of markups in canadian manufacturing: A production theory approach, Journal of Applied Econometrics 9, 269-282.

Pindyck, R.S. and J.J. Rotemberg, 1983, Dynamic factor demands and the effects of energy price shocks, American Economic Review 73(5), 1066-1079.

Reynolds, S.S., 1987, Capacity investment, preemption and commitment in an infinite horizon model, International Economic Review 28(1), 69-88.

Tirole, J., 1988, The theory of industrial organization (MIT Press, Cambridge M.).

Ulph, A., 1992, The choice of environmental policy instruments and strategic international trade, in: R. Pethig, ed., Conflicts and cooperation in managing environmental resources (Springer-Verlag, Berlin).

Ulph, A., 1996, Environmental policy instruments and imperfectly competitive international trade, Environmental and Resource Economics 7, 333-355.